INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

SENIOR PAPER: YEARS 11,12

Tournament 40, Northern Spring 2019 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. The distances from some point inside a regular hexagon to three of its vertices that are consecutive, are equal to 1, 1 and 2, respectively. Determine the side length of the hexagon. (4 points)

2. Let a and b be positive integers such that $a^{n+1} + b^{n+1}$ is divisible by $a^n + b^n$ for infinitely many positive integers n. Is it necessarily true that $a = b$? (4 points)

3. Prove that any triangle can be cut into 2019 quadrilaterals each of which has both a circumcircle (i.e. is cyclic) and an incircle (i.e. a circle that touches each of the quadrilateral's four sides). (4 points)

4. A magician and his assistant present the following trick. Thirteen empty closed boxes are placed in a row. Then, the magician leaves the stage, and a random person from the audience is selected to put two coins into two boxes of their choice, one coin in each box, in front of the magician's assistant, i.e. the assistant knows which boxes contain coins. Then, the magician returns and his assistant is allowed to open one box that does not contain a coin. After that the magician must choose four boxes to be opened simultaneously. The goal of the magician is to open both boxes with coins. Construct a scheme by which the magician and his assistant can perform the trick successfully every time. (5 points)

5. Several positive integers that sum to 2019, are written in sequence. Neither a number in the sequence, nor a sum of two or more adjacent numbers is equal to 40. What is the largest number of positive integers the sequence could consist of? (5 points)